

**Review 3, No Calculator**

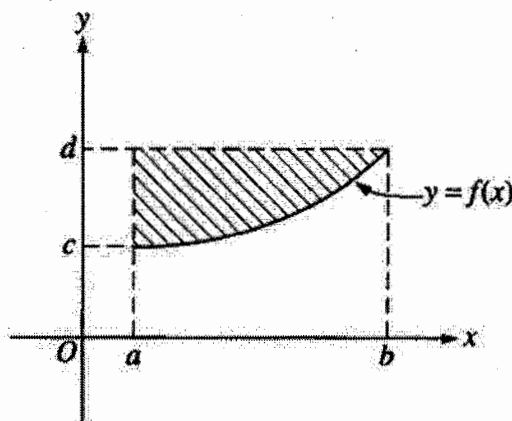
Complete all the following on notebook paper.

\_\_\_\_ 1.

If  $f(x) = x^{\frac{3}{2}}$ , then  $f'(4) =$ 

- (A) -6 (B) -3 (C) 3 (D) 6 (E) 8

\_\_\_\_ 2.



Which of the following represents the area of the shaded region in the figure above?

- (A)
- $\int_c^d f(y) dy$
- (B)
- $\int_a^b (d - f(x)) dx$
- (C)
- $f'(b) - f'(a)$
- 
- (D)
- $(b - a)[f(b) - f(a)]$
- (E)
- $(d - c)[f(b) - f(a)]$

\_\_\_\_ 3.

 $\lim_{n \rightarrow \infty} \frac{3n^3 - 5n}{n^3 - 2n^2 + 1}$  is

- (A) -5 (B) -2 (C) 1 (D) 3 (E) nonexistent

\_\_\_\_ 4.

If the function  $f$  is continuous for all real numbers and if  $f(x) = \frac{x^2 - 4}{x + 2}$  when  $x \neq -2$ , then  $f(-2) =$ 

- (A) -4 (B) -2 (C) -1 (D) 0 (E) 2

5.

If  $x^3 + 3xy + 2y^3 = 17$ , then in terms of  $x$  and  $y$ ,  $\frac{dy}{dx} =$

(A)  $-\frac{x^2 + y}{x + 2y^2}$

(B)  $-\frac{x^2 + y}{x + y^2}$

(C)  $-\frac{x^2 + y}{x + 2y}$

(D)  $-\frac{x^2 + y}{2y^2}$

(E)  $\frac{-x^2}{1 + 2y^2}$

6.

The area of the region enclosed by the curve  $y = \frac{1}{x-1}$ , the  $x$ -axis, and the lines  $x = 3$  and  $x = 4$  is

(A)  $\frac{5}{36}$

(B)  $\ln \frac{2}{3}$

(C)  $\ln \frac{4}{3}$

(D)  $\ln \frac{3}{2}$

(E)  $\ln 6$

7.

An equation of the line tangent to the graph of  $y = \frac{2x+3}{3x-2}$  at the point  $(1, 5)$  is

(A)  $13x - y = 8$

(B)  $13x + y = 18$

(C)  $x - 13y = 64$

(D)  $x + 13y = 66$

(E)  $-2x + 3y = 13$

8.

If  $y = \tan x - \cot x$ , then  $\frac{dy}{dx} =$

(A)  $\sec x \csc x$

(B)  $\sec x - \csc x$

(C)  $\sec x + \csc x$

(D)  $\sec^2 x - \csc^2 x$

(E)  $\sec^2 x + \csc^2 x$

9.

If  $h$  is the function given by  $h(x) = f(g(x))$ , where  $f(x) = 3x^2 - 1$  and  $g(x) = |x|$ , then  $h(x) =$

- (A)  $3x^3 - |x|$       (B)  $|3x^2 - 1|$       (C)  $3x^2|x| - 1$       (D)  $3|x| - 1$       (E)  $3x^2 - 1$

10.

If  $f(x) = (x-1)^2 \sin x$ , then  $f'(0) =$

- (A)  $-2$       (B)  $-1$       (C)  $0$       (D)  $1$       (E)  $2$

11. 2001—AB5

A cubic polynomial function  $f$  is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where  $a$ ,  $b$ , and  $k$  are constants. The function  $f$  has a local minimum at  $x = -1$ , and the graph of  $f$  has a point of inflection at  $x = -2$ .

(a) Find the values of  $a$  and  $b$ .

(b) If  $\int_0^1 f(x) dx = 32$ , what is the value of  $k$ ?

12. 2001—AB6

The function  $f$  is differentiable for all real numbers. The point  $\left(3, \frac{1}{4}\right)$  is on the graph of  $y = f(x)$ , and the slope at each point  $(x, y)$  on the graph is given by  $\frac{dy}{dx} = y^2(6 - 2x)$ .

(a) Find  $\frac{d^2y}{dx^2}$  and evaluate it at the point  $\left(3, \frac{1}{4}\right)$ .

(b) Find  $y = f(x)$  by solving the differential equation  $\frac{dy}{dx} = y^2(6 - 2x)$  with the initial condition  $f(3) = \frac{1}{4}$ .

# Review 3 solutions

$$1. f(x) = x^{3/2}$$

$$f'(x) = \frac{3}{2} x^{1/2}$$

$$f'(4) = \frac{3}{2} (4)^{1/2} = 3 \quad \boxed{C}$$

$$2. \int_a^b (d - f(x)) dx \quad \boxed{B}$$

$$3. \lim_{n \rightarrow \infty} \frac{3n^3 - 5n}{n^3 - 2n^2 + 1} = \frac{3}{1} \quad \boxed{D}$$

$$4. f(x) = \frac{x^2 - 4}{x + 2} = \frac{(x+2)(x-2)}{x+2}$$

$$f(-2) = (-2-2) = -4 \quad \boxed{A}$$

$$5. 3x^2 + 3x \frac{dy}{dx} + 3y + 6y^2 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-3x^2 - 3y}{3x + 6y^2} = \frac{-x^2 - y}{x + 2y^2} = -\frac{x^2 + y}{x + 2y^2} \quad \boxed{C}$$

$$6. \int_2^4 \frac{1}{x-1} dx = \ln|x-1| \Big|_2^4$$

$$\ln 3 - \ln 2 = \ln \frac{3}{2} \quad \boxed{D}$$

$$7. y = \frac{2x+3}{3x-2}$$

$$y' = \frac{(3x-2)(2) - (2x+3)(3)}{(3x-2)^2}$$

$$y' = \frac{6x-4-6x-9}{(3x-2)^2} = \frac{-13}{(3x-2)^2}$$

$$y'(1) = \frac{-13}{1} \rightarrow y-5 = -13(x-1)$$

$$y = -13x + 18 \quad \boxed{B}$$

$$8. \frac{dy}{dx} = \sec^2 x + \csc^2 x \quad \boxed{E}$$

a.  $h(x) = f(g(x))$

$$= f(|x|)$$

$$= 3(|x|)^2 - 1 \quad \boxed{E}$$

10.  $f'(x) = 2(x-1)\sin x + (x-1)^2 \cos x$

$$f'(0) = 2(-1)\sin 0 + (-1)^2 \cos 0$$

$$= -2(0) + 1(1) = 1 \quad \boxed{D}$$

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**Question 5**

A cubic polynomial function  $f$  is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where  $a$ ,  $b$ , and  $k$  are constants. The function  $f$  has a local minimum at  $x = -1$ , and the graph of  $f$  has a point of inflection at  $x = -2$ .

(a) Find the values of  $a$  and  $b$ .

(b) If  $\int_0^1 f(x) dx = 32$ , what is the value of  $k$ ?

(a)  $f'(x) = 12x^2 + 2ax + b$

$$f''(x) = 24x + 2a$$

$$f'(-1) = 12 - 2a + b = 0$$

$$f''(-2) = -48 + 2a = 0$$

$$a = 24$$

$$b = -12 + 2a = 36$$

$$5 : \left\{ \begin{array}{l} 1 : f'(x) \\ 1 : f''(x) \\ 1 : f'(-1) = 0 \\ 1 : f''(-2) = 0 \\ 1 : a, b \end{array} \right.$$

(b)  $\int_0^1 (4x^3 + 24x^2 + 36x + k) dx$   
 $= x^4 + 8x^3 + 18x^2 + kx \Big|_{x=0}^{x=1} = 27 + k$

$$27 + k = 32$$

$$k = 5$$

$$4 : \left\{ \begin{array}{l} 2 : \text{antidifferentiation} \\ \quad < -1 > \text{ each error} \\ 1 : \text{expression in } k \\ 1 : k \end{array} \right.$$

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**Question 6**

The function  $f$  is differentiable for all real numbers. The point  $\left(3, \frac{1}{4}\right)$  is on the graph of  $y = f(x)$ , and the slope at each point  $(x, y)$  on the graph is given by  $\frac{dy}{dx} = y^2(6 - 2x)$ .

- (a) Find  $\frac{d^2y}{dx^2}$  and evaluate it at the point  $\left(3, \frac{1}{4}\right)$ .
- (b) Find  $y = f(x)$  by solving the differential equation  $\frac{dy}{dx} = y^2(6 - 2x)$  with the initial condition  $f(3) = \frac{1}{4}$ .

(a) 
$$\begin{aligned}\frac{d^2y}{dx^2} &= 2y \frac{dy}{dx} (6 - 2x) - 2y^2 \\ &= 2y^3(6 - 2x)^2 - 2y^2\end{aligned}$$

$$\left. \frac{d^2y}{dx^2} \right|_{\left(3, \frac{1}{4}\right)} = 0 - 2\left(\frac{1}{4}\right)^2 = -\frac{1}{8}$$

(b) 
$$\frac{1}{y^2} dy = (6 - 2x) dx$$

$$-\frac{1}{y} = 6x - x^2 + C$$

$$-4 = 18 - 9 + C = 9 + C$$

$$C = -13$$

$$y = \frac{1}{x^2 - 6x + 13}$$

$$3 : \begin{cases} 2 : \frac{d^2y}{dx^2} \\ < -2 > \text{product rule or} \\ &\text{chain rule error} \\ 1 : \text{value at } \left(3, \frac{1}{4}\right) \end{cases}$$

$$6 : \begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivative of } dy \text{ term} \\ 1 : \text{antiderivative of } dx \text{ term} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition } f(3) = \frac{1}{4} \\ 1 : \text{solves for } y \end{cases}$$

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables